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Dynamic effective properties of the particle-reinforced composites with the viscoelastic interphase

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Abstract

The effects of viscoelastic behavior of the interphase on the dynamic effective properties of composite materials reinforced by the distributed coated spherical inclusions are studied in this paper. The effective wave numbers of composites are predicted from the coherent plane wave equation which is related to the forward scattering amplitudes of an individual inclusion. A thin homogeneous viscoelastic interphase between the inclusion and the matrix is used to model the more realistic bonding state between them. Because the forward scattering amplitudes are closely related to the interphase, the interphase thus can affect the effective properties of composites significantly. The numerical simulation is given for SiC–Al composites and it is shown that the effective wave numbers and the effective elastic moduli of the composites are affected by the viscosity of the interphase noticeably. The attenuation of the effective waves is related to both the multiple scattering amongst reinforced particles and the material dissipation of the viscoelastic interphase. However, the dissipation effect of the interphase dominates in a range of relatively low frequency, whereas the effect of multiple scattering dominates in a range of relatively high frequency.

Keywords: Viscoelastic interphase; Multiple scattering; Dynamic effective properties; Composites

1. Introduction

The determination of the effective propagation constants of the waves propagating through composite materials has been a subject which attracted a considerable attention in the past several decades (e.g. Foldy, 1945; Lax, 1952; Varadan et al., 1985; Data et al., 1988; Shindo et al., 1995; Kanaun, 2000). Foldy (1945) studied early the effective wave number of the scalar wave propagating through the inhomogeneous medium with distributed particles based on the multiple scattering theory. In this theory a set of equations

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in hierarchy, each containing more statistical information than those preceding, is involved. Truncating these equations to obtain an approximate solution usually resorts to the well-known "quasi-crystalline approximation" proposed by Lax (1952). Later, Bose and Mal (1973) and Varadan et al. (1985) extended the multiple scattering theory of the scalar wave to the elastic waves and enhanced the theory by introducing the more realistic pair-correlation function to describe the interaction between two particles accurately. On the other hand, the interaction amongst particles can be described approximately by assuming that each particle is embedied in an effective medium, which is usually called as the effective medium approach and was employed by Berryman (1980), Sabina and Willis (1988), Yang and Mal (1994), Kanaun (2000) and others. The multiple scattering theory and the effective medium approach are based on different assumptions to simplify calculations, and thus, generally speaking, will give different results when applied to a given composite medium.

In the composites reinforced by fibers or particles, it is often the case that there is an imperfect interface between the matrix and the fiber or particle induced by processing conditions. The nondestructive characterizing of interface properties by ultrasonic waves is crucial for the safety service of structure material. Consequently, it is desirable to relate the effective propagation constants (the phase velocity and the attenuation) to the properties of the imperfect interface. Mal and Bose (1974) studied early the imperfect interface where only the tangential displacement jumps were considered. This means the slip may occur at the interface if a load is applied on it. Data et al. (1988) studied the imperfect interface that both tangential and normal displacements jumps exist. In both studies above-mentioned it is assumed that the tractions are continuous across the interface. It may be noticed that these approximate boundary conditions ignore the inertial and the curvature effects. The imperfect interface with both displacement and stress jumps, as an improved model, was studied by Olsson et al. (1990) and Hashin (2002). Further, the graded interfacial layer in the fiber- or particle-reinforced composites was discussed by Shindo et al. (1995) and Sato and Shindo (2001).

In the present work the interphase is modeled as a thin shell with finite thickness and is assumed to be viscoelastic. An interphase of such nature might be introduced to provide relaxation and damping characteristics to an otherwise elastic brittle composite. The effects of viscoelastic interphase on the effective properties of composites were studied by Hashin (1991) based on the correspondence principle. In his investigation, the Maxwell model of viscoelastic material is used. The effects of viscoelastic matrix and viscoelastic particle were studied recently by Biwa et al. (2002) based on the independent scattering/absorption analysis, but the complex moduli of the viscoelastic material were approximated as frequency-independent constants. It is our purpose to discuss the effects of the viscoelastic interphase on the effective propagation constants and the dynamic effective moduli of such composites. And the more general model for a viscoelastic material, i.e. standard solid model, will be used in our study. The outline of the paper is as follows: In Section 2, the scattering problem of a single inclusion embedded in an elastic matrix with the viscoelastic interphase separating the inclusion with the matrix is studied and the forward scattering amplitudes are formulated. In Section 3, an equation to predict the effective wave number by using the forward scattering amplitudes of displacement vector is formulated. And some other equations to predict the effective wave number by using the forward scattering amplitudes of wave potential are discussed. In Section 4, the effects of the viscoelastic interphase on the effective properties of composites are studied and the numerical calculations are carried out for SiC–Al composites. Finally, some conclusions are given in Section 5.

2. Scattered waves by a coated spherical particle embedded in a elastic matrix

Consider a spherical inclusion of radius a embedded in an elastic matrix. The *lam'e* constants and the mass densities of the inclusion and the matrix are denoted by $(\lambda_1, \mu_1, \rho_1)$ and $(\lambda_0, \mu_0, \rho_0)$, respectively. We assume that the inclusion is separated from the matrix by a thin viscoelastic interphase of uniform thickness h. The frequency-dependent *lam'e* constants and the mass densities of the viscoelastic interphase are denoted by $(\lambda_2^v(\omega), \mu_2^v(\omega), \rho_2)$. The geometry is depicted in Fig. 1, where (x, y, z) is the right-handed Cartesian coordinate system with the origin at the center of the spherical inclusion and (r, θ, ϕ) is the corresponding spherical polar coordinate. The time harmonic plane longitudinal and shear waves, P and S waves, with circular frequency ω are assumed to propagate through the matrix. Let the z-axis is the propagation direction of the incident waves. Then, the incident waves may be expressed by the displacement vector

$$\mathbf{u}^i = \mathbf{a}e^{i(k_{p0}z-\omega t)} + \mathbf{b}e^{i(k_{s0}z-\omega t)}, \tag{1}$$

where $\mathbf{a} = a\mathbf{e}_z$ and $\mathbf{b} = b\mathbf{e}_x$ are the polarization vectors of incident P and S waves, respectively. $\mathbf{e}_z$ and $\mathbf{e}_x$ are unit coordinate vectors. k_{p0} and k_{s0} are the wave numbers of the incident P and S waves, respectively. When the incident waves impinge the coated elastic inclusion, the scattered waves outside the coated inclusion, the refracted waves inside the inclusion and the transmitted waves in the interphase are induced. It is no doubt that the existence of the interphase can change the scattered waves and thus affect the effective waves propagating through the composites.

In order to evaluate the scattered wave field in the matrix, it is necessary to take into account the transmitted waves in the interphase and the continuous conditions of displacements and tractions across the interfaces at both sides of the interphase. It is noted that the wave numbers of waves in the elastic inclusion and matrix are real-valued and the wave numbers of waves propagating through the interphase are complex-valued due to the frequency-dependent complex moduli of the viscoelastic interphase. The complex-value wave numbers mean the attenuation of waves, in other word, the energy carried by waves is partly absorbed by the viscoelastic interphase. This mechanism of the viscoelastic interphase is expected to improve the mechanical properties of brittle composites. The constitutive equations of an isotropic viscoelastic material can be expressed generally in the Stieltjes integral form

$$s_{ij}(t) = \int_{-\infty}^{t} 2G(\tau)\dot{e}_{ij}(t-\tau)\,d\tau, \tag{2a}$$

$$\sigma_{kk}(t) = \int_{-\infty}^{t} 3K(\tau)\dot{\varepsilon}_{kk}(t-\tau)\,d\tau. \tag{2b}$$

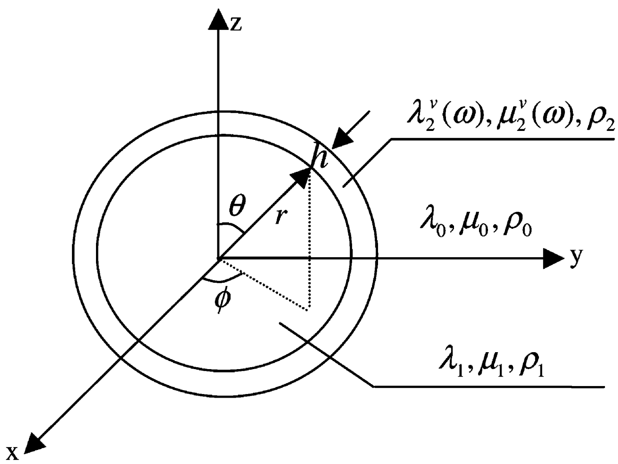

Fig. 1. A coated spherical inclusion embedded in an elastic matrix.

In the harmonic cases, it leads to

$$\bar{s}_{ij}(\omega) = 2G^v(\omega)\bar{e}_{ij}(\omega), \tag{3a}$$

$$\bar{\sigma}_{kk}(\omega) = 3K^v(\omega)\bar{\varepsilon}_{kk}(\omega), \tag{3b}$$

where ε_{ij}, σ_{ij}, e_{ij}, s_{ij} are the strain, the stress, the deviatoric strain and the deviatoric stress, respectively. And $\bar{\varepsilon}_{ij}$, $\bar{\sigma}_{ij}$, $\bar{e}_{ij}$, $\bar{s}_{ij}$ are their corresponding Fourier transformations, respectively, i.e.$\bar{f}(\omega) = F(f(t))$. The shear and bulk moduli, $G^v(\omega)$ and $K^v(\omega)$, are related to the shear and bulk relaxation functions, $G(t)$ and $K(t)$, by

$$G^v(\omega) = \mathrm{i}\omega F(G(t)), \tag{4a}$$

$$K^v(\omega) = \mathrm{i}\omega F(K(t)). \tag{4b}$$

The equations of wave motion in a homogeneous elastic or viscoelastic medium are expressed as

$$k_p^{-2}\nabla\nabla\cdot\mathbf{u} - k_s^{-2}\nabla\times\nabla\times\mathbf{u} + \mathbf{u} = 0, \tag{5}$$

where $\mathbf{u}(x, y, z, t)$ is the time harmonic displacement vector. For convenience, the time harmonic factor $\mathrm{e}^{-\mathrm{i}\omega t}$ is omitted in the following discussion but understood. $k_p = \omega/\sqrt{(\lambda+2\mu)/\rho}$ and $k_s = \omega/\sqrt{\mu/\rho}$ are the wave numbers (real- or complex-value) of the longitudinal and the shear waves, respectively. ∇ is the gradient operator. It is known that the general form of the solution of Eq. (5) can be expressed as

$$\mathbf{u} = \nabla\Phi + \nabla\times(\Psi r)\mathbf{e_r} + \nabla\times\nabla\times(\Pi r)\mathbf{e_r}, \tag{6}$$

where the scalar potential Φ, Ψ and Π are the solutions of the scalar Helmholtz equation

$$(\nabla^2 + k^2)(\Phi, \Psi, \Pi) = 0, \tag{7}$$

(where ∇^2 is the Laplacian operator) and can be expressed in a series form

$$(\Phi, \Psi, \Pi) = \sum_{n=0}^{\infty}\sum_{m=0}^{\pm n} C_{nm} Z_n^q(kr) P_n^m(\cos\theta)\mathrm{e}^{\mathrm{i}m\phi} \quad (q = 1 \text{ or } 3), \tag{8}$$

where C_{nm} is the expansion coefficient. $P_n^m(\cos\theta)$ is the associated Legendre function and the symbol $Z_n^q(kr)$ stands for the spherical Bessel function $j_n(kr)$ for $q = 1$ and the spherical Hankel function $h_n^{(1)}(kr)$ for $q = 3$. In order to meet the radial conditions at infinity and to keep finite values of the displacements at the center of the inclusion, the potentials of the scattered, transmitted and refracted waves can be expressed as

$$\Phi^s = \sum_{n=0}^{\infty}\sum_{m=0}^{\pm n} A_{mn}^s h_n^{(1)}(k_p r) P_n^m(\cos\theta)\mathrm{e}^{\mathrm{i}m\phi}, \tag{9a}$$

$$\Psi^s = \sum_{n=0}^{\infty}\sum_{m=0}^{\pm n} B_{mn}^s h_n^{(1)}(k_s r) P_n^m(\cos\theta)\mathrm{e}^{\mathrm{i}m\phi}, \tag{9b}$$

$$\Pi^s = \sum_{n=0}^{\infty}\sum_{m=0}^{\pm n} C_{mn}^s h_n^{(1)}(k_s r) P_n^m(\cos\theta)\mathrm{e}^{\mathrm{i}m\phi}, \tag{9c}$$

$$\Phi^t = \sum_{n=0}^{\infty}\sum_{m=0}^{\pm n} [\overline{A}_{mn}^t j_n(k_p r) P_n^m(\cos\theta) + A_{mn}^t h_n^{(1)}(k_p r) P_n^m(\cos\theta)]\mathrm{e}^{\mathrm{i}m\phi}, \tag{10a}$$

$$\Psi^t = \sum_{n=0}^{\infty}\sum_{m=0}^{\pm n}[\overline{B}^t_{mn} j_n(k_s r) P_n^m(\cos\theta) + B^t_{mn} h_n^{(1)}(k_s r) P_n^m(\cos\theta)]\mathrm{e}^{\mathrm{i}m\phi}, \tag{10b}$$

$$\Pi^t = \sum_{n=0}^{\infty}\sum_{m=0}^{\pm n}[\overline{C}^t_{mn} j_n(k_s r) P_n^m(\cos\theta) + C^t_{mn} h_n^{(1)}(k_s r) P_n^m(\cos\theta)]\mathrm{e}^{\mathrm{i}m\phi}, \tag{10c}$$

$$\Phi^r = \sum_{n=0}^{\infty}\sum_{m=0}^{\pm n} A^r_{mn} j_n(k_p r) P_n^m(\cos\theta)\mathrm{e}^{\mathrm{i}m\phi}, \tag{11a}$$

$$\Psi^r = \sum_{n=0}^{\infty}\sum_{m=0}^{\pm n} B^r_{mn} j_n(k_s r) P_n^m(\cos\theta)\mathrm{e}^{\mathrm{i}m\phi}, \tag{11b}$$

$$\Pi^r = \sum_{n=0}^{\infty}\sum_{m=0}^{\pm n} C^r_{mn} j_n(k_s r) P_n^m(\cos\theta)\mathrm{e}^{\mathrm{i}m\phi}, \tag{11c}$$

where A_n^α, B_n^α, C_n^α, $\overline{A}_n^\alpha$, $\overline{B}_n^\alpha$ and $\overline{C}_n^\alpha$ ($\alpha = s, r, t$ for the scattered, refracted and transmitted waves, respectively) are the expansion coefficients to be determined from the boundary conditions. The boundary conditions, namely, displacements and tractions are continuous across the interfaces at both sides of the interphase, may be written as

$$u^r_\beta(a) = u^t_\beta(a), \quad u^i_\beta(a+h) + u^s_\beta(a+h) = u^t_\beta(a+h) \quad (\beta = r, \theta, \phi), \tag{12a}$$

$$t^r_\beta(a) = t^t_\beta(a), \quad t^i_\beta(a+h) + t^s_\beta(a+h) = t^t_\beta(a+h) \quad (\beta = r, \theta, \phi). \tag{12b}$$

where the tractions vector can be obtained by

$$\mathbf{t} = \boldsymbol{\sigma}\cdot\mathbf{n}, \tag{13}$$

$$\boldsymbol{\sigma} = \lambda(\nabla\cdot\mathbf{u})\mathbf{I} + \mu(\nabla\mathbf{u} + \mathbf{u}\nabla). \tag{14}$$

In Eq. (13) and (14), $\mathbf{n}$ is the outward normal vector of the interfaces and $\mathbf{I}$ is the second-rank identity tensor. In order to determine the unknown expansion constants, it is convenient to extend the incident plane waves in terms of the spherical wave functions too

$$\begin{aligned}\Phi^i &= \frac{a}{\mathrm{i}k_{p0}}\mathrm{e}^{\mathrm{i}k_{p0}z} = \frac{a}{\mathrm{i}k_{p0}}\sum_{n=0}^{\infty}\sum_{m=0}^{\pm n}(2n+1)\mathrm{i}^n\delta_{m0} j_n(k_{p0}r)P_n^m(\cos\theta)\mathrm{e}^{\mathrm{i}m\phi} \\ &= \sum_{n=0}^{\infty}\sum_{m=0}^{\pm n} A^i_{mn} j_n(k_{p0}r) P_m^n(\cos\theta)\mathrm{e}^{\mathrm{i}m\phi},\end{aligned} \tag{15a}$$

$$\begin{aligned}\Psi^i &= \frac{b}{\mathrm{i}k_{s0}}\sum_{n=0}^{\infty}\sum_{m=0}^{\pm n}\frac{2n+1}{2n(n+1)}[\delta_{m,1} + n(n+1)\delta_{m,-1}]\mathrm{i}^{n-1} j_n(k_{s0}r)P_n^m(\cos\theta)\mathrm{e}^{\mathrm{i}m\phi} \\ &= \sum_{n=0}^{\infty}\sum_{m=0}^{\pm n} B^i_{mn} j_n(k_{s0}r) P_m^n(\cos\theta)\mathrm{e}^{\mathrm{i}m\phi},\end{aligned} \tag{15b}$$

$$\Pi^i = \frac{b}{(\mathrm{i}k_{s0})^2}\sum_{n=0}^{\infty}\sum_{m=0}^{\pm n}\frac{2n+1}{2n(n+1)}[\delta_{m,1} - n(n+1)\delta_{m,-1}]\mathrm{i}^{n-1}j_n(k_{s0}r)P_n^m(\cos\theta)\mathrm{e}^{\mathrm{i}m\phi}$$
$$= \sum_{n=0}^{\infty}\sum_{m=0}^{\pm n} C^i_{mn} j_n(k_{s0}r)P^n_m(\cos\theta)\mathrm{e}^{\mathrm{i}m\phi}. \tag{15c}$$

The solution of Eq. (12) can be expressed formally

$$(A^s_{mn}, B^s_{mn}, C^s_{mn}, A^t_{mn}, B^t_{mn}, C^t_{mn}, \overline{A}^t_{mn}, \overline{B}^t_{mn}, \overline{C}^t_{mn}, A^r_{mn}, B^r_{mn}, C^r_{mn})^r = \mathbf{T}\cdot(A^i_{mn}, B^i_{mn}, C^i_{mn})^t. \tag{16}$$

where $\mathbf{T}$ is the so-called T-matrix which is dependent upon the properties of the matrix, the inclusion and the interphase between them. Furthermore, with the introduction of the scattering operator $\mathbf{T}^s$, the scattered wave can be related to the incident wave by

$$\mathbf{u}^s = \mathbf{T}^s\mathbf{u}^i. \tag{17}$$

After applying the asymptotic expression of the radial function $h_n^{(1)}(kr)$

$$h_n^{(1)}(kr) \sim \frac{1}{kr}\mathrm{e}^{\mathrm{i}[kr-\frac{1}{2}(n+1)\pi]} + \mathrm{o}\left(\frac{1}{r}\right) \quad \text{when } r\to\infty, \tag{18}$$

the displacement of scattered wave in the far-field can be expressed asymptotically

$$u_r \sim \frac{1}{r}\mathrm{e}^{\mathrm{i}k_{p0}r}\sum_{n=0}^{\infty}\sum_{m=0}^{\pm n}\mathrm{i}A^s_{mn}\mathrm{e}^{-\mathrm{i}\frac{1}{2}(n+1)\pi}P_n^m(\cos\theta)\mathrm{e}^{\mathrm{i}m\phi} + \mathrm{o}\left(\frac{1}{r}\right) = \frac{F_r(\theta,\phi)}{r}\mathrm{e}^{\mathrm{i}k_{p0}r} + \mathrm{o}\left(\frac{1}{r}\right), \tag{19a}$$

$$u_\theta \sim \frac{1}{r}\mathrm{e}^{\mathrm{i}k_{s0}r}\sum_{n=0}^{\infty}\sum_{m=0}^{\pm n}\mathrm{i}\mathrm{e}^{-\mathrm{i}\frac{1}{2}(n+1)\pi}\left[B^s_{mn}\frac{m}{k_{s0}\sin\theta}P_n^m(\cos\theta) + C^s_{mn}\frac{d}{d\theta}P_n^m(\cos\theta)\right]\mathrm{e}^{\mathrm{i}m\phi} + \mathrm{o}\left(\frac{1}{r}\right)$$
$$= \frac{F_\theta(\theta,\phi)}{r}\mathrm{e}^{\mathrm{i}k_{s0}r} + \mathrm{o}\left(\frac{1}{r}\right), \tag{19b}$$

$$u_\phi \sim -\frac{1}{r}\mathrm{e}^{\mathrm{i}k_{s0}r}\sum_{n=0}^{\infty}\sum_{m=0}^{\pm n}\left[\frac{B_{mn^s}}{k_{s0}}\frac{d}{d\theta}P_n^m(\cos\theta) + \frac{m}{\sin\theta}C^s_{mn}P_n^m(\cos\theta)\right]\mathrm{e}^{-\mathrm{i}\frac{1}{2}(n+1)\pi}\mathrm{e}^{\mathrm{i}m\phi} + \mathrm{o}\left(\frac{1}{r}\right)$$
$$= \frac{F_\theta(\theta,\phi)}{r}\mathrm{e}^{\mathrm{i}k_{s0}r} + \mathrm{o}\left(\frac{1}{r}\right), \tag{19c}$$

where $F_r(\theta,\phi)$, $F_\theta(\theta,\phi)$ and $F_\phi(\theta,\phi)$ are called the far-field scattered amplitudes of displacement components. Furthermore, we define $\mathbf{F}_p = F_r(\theta,\phi)\mathbf{e}_r$ and $\mathbf{F}_s = F_\theta(\theta,\phi)\mathbf{e}_\theta + F_\phi(\theta,\phi)\mathbf{e}_\phi$ ($\mathbf{e}_r$, $\mathbf{e}_\theta$ and $\mathbf{e}_\phi$ is unit polar coordinate vectors) as the far-field scattered amplitude vectors for the scattered longitudinal and shear waves, respectively. It is noted that the far-field scattered amplitudes are dependent on the azimuth angles (θ,ϕ). The far-field scattered amplitudes at two specific azimuthal angles, $\theta = 0$ and $\theta = \pi$, are of special interest, and are called the forward and the backward scattering amplitudes, respectively.

$$F_r(0,\phi) = \sum_{n=0}^{\infty}(-\mathrm{i})^n A^s_{0n}, \tag{20a}$$

$$F_\theta(0,\phi) = \sum_{n=1}^{\infty}\frac{(-\mathrm{i})^n}{2}\left[\frac{n(n+1)}{k_{s0}}B^s_{1n}\mathrm{e}^{\mathrm{i}\phi} + \frac{1}{k_{s0}}B^s_{-1n}\mathrm{e}^{-\mathrm{i}\phi}\right], \tag{20b}$$

$$F_\phi(0,\phi) = \sum_{n=1}^{\infty} \frac{\mathrm{i}(-\mathrm{i})^n}{2}[n(n+1)C^s_{1n}\mathrm{e}^{\mathrm{i}\phi} + C^s_{-1n}\mathrm{e}^{-\mathrm{i}\phi}], \tag{20c}$$

$$F_r(\pi,\phi) = \sum_{n=0}^{\infty} \mathrm{i}^n A^s_{0n}, \tag{21a}$$

$$F_\theta(\pi,\phi) = \sum_{n=1}^{\infty} \frac{-\mathrm{i}^n}{2}\left[\frac{n(n+1)}{k_{s0}}B^s_{1n}\mathrm{e}^{\mathrm{i}\phi} + \frac{1}{k_{s0}}B^s_{-1n}\mathrm{e}^{-\mathrm{i}\phi}\right], \tag{21b}$$

$$F_\phi(\pi,\phi) = \sum_{n=1}^{\infty} \frac{-\mathrm{i}^{n+1}}{2}[n(n+1)C^s_{1n}\mathrm{e}^{\mathrm{i}\phi} + C^s_{-1n}\mathrm{e}^{-\mathrm{i}\phi}]. \tag{21c}$$

3. Dynamic effective properties of the particle-reinforced composites

We now consider a composite material with N inclusions randomly distributed in the matrix. If their positions of these inclusions, denoted by the random variables $(\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_N)$, are given, we shall say that we have a particular configuration of these scatterers. The joint probabilities distribution, denoted by $p(\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_N)$, represents the probability of finding these scatterers in the above configuration. In light of the chain rule of the conditional probabilities, the distribution function can be written as

$$p(\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_N) = p(\mathbf{r}_i)p(\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_{i-1}, \mathbf{r}_{i+1}, \ldots, \mathbf{r}_N|\mathbf{r}_i) = p(\mathbf{r}_i)p(\mathbf{r}_1, \mathbf{r}_2, \ldots{}' \ldots, \mathbf{r}_N|\mathbf{r}_i), \tag{22}$$

where the vertical lines in the arguments stands for the conditional probability distribution with the scatterer positioned at $\mathbf{r}_i$ hold fixed. Symbol "′" means the absence of one variable. Due to the indistinguishability of inclusions, the distribution function $p(\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_N)$ is symmetric in its arguments. If the composite medium is statistically uniform within a volume V, then, the position of each inclusion is equally probable within the volume V, namely, its distribution is uniform with density

$$p(\mathbf{r}_i) = \frac{1}{V} \quad (i = 1, 2, 3, \ldots, N). \tag{23}$$

The probability of finding a particular inclusion in the micro volume element $\mathrm{d}V_i$ at $\mathbf{r}_i$ is

$$p(\mathbf{r}_i)\,\mathrm{d}V_i = \mathrm{d}V_i \int \cdots \int \mathrm{d}V_1 \cdots{}' \cdots \mathrm{d}V_N p(\mathbf{r}_1, \ldots, \mathbf{r}_N). \tag{24}$$

Since each of the N inclusions has equal likelihood for occupying $\mathrm{d}V_i$, the number density $n(\mathbf{r}_i)$ of inclusions at $\mathbf{r}_i$ is then given by

$$n(\mathbf{r}_i) = Np(\mathbf{r}_i), \tag{25}$$

and is related to the volume concentration c by $n = 3c/(4\pi a^3)$. The configurational average of a random function $f(\mathbf{r}; \mathbf{r}_1, \ldots, \mathbf{r}_N)$ is defined by

$$\langle f(\mathbf{r}; \mathbf{r}_1, \ldots, \mathbf{r}_N)\rangle = \int \cdots \int \mathrm{d}V_1 \cdots \mathrm{d}V_N p(\mathbf{r}_1, \ldots, \mathbf{r}_N) f(\mathbf{r}; \mathbf{r}_1, \ldots, \mathbf{r}_N), \tag{26}$$

and the partial configurational average with one inclusion held fixed is defined by

$$\langle f(\mathbf{r}|\mathbf{r}_i; \mathbf{r}_1, \ldots{}' \ldots, \mathbf{r}_N)\rangle = \int \cdots \int \mathrm{d}V_1 \ldots{}' \ldots \mathrm{d}V_N p(\mathbf{r}_1, \ldots{}' \ldots, \mathbf{r}_N|\mathbf{r}_i) f(\mathbf{r}|\mathbf{r}_i; \mathbf{r}_1, \ldots{}' \ldots, \mathbf{r}_N), \tag{27}$$

where the first coordinate $\mathbf{r}$ indicates the field point of evaluation, and the $(\mathbf{r}_1, \ldots, \mathbf{r}_N)$ indicates the dependence of the random function on the specific configuration chosen.

The total field at any point outside all inclusions can be given in the multiple scattering form

$$\mathbf{u}(\mathbf{r}; \mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_N) = \mathbf{u}^i(\mathbf{r}) + \sum_{k=1}^{N} \mathbf{T}^s(\mathbf{r}_k)\mathbf{u}^i(\mathbf{r}) + \sum_{m=1}^{N} \mathbf{T}^s(\mathbf{r}_m) \sum_{k=1,k\neq m}^{N} \mathbf{T}^s(\mathbf{r}_k)\mathbf{u}^i(\mathbf{r}) + \cdots, \tag{28}$$

where the single summation denotes the primary scattered terms, the double summation the secondary terms and so on. The primary scattering is due to the incident waves alone, and the second scattering represents the rescattering of the primary scattered waves, etc. The multiple scattering theory takes into account the interaction among the distributed inclusions accurately. However, it is difficulty to deal with in order to predict the effective properties. Here, we use the effective field approximation to describe approximately the interaction among the distributed inclusions. In this approximation it is assumed that each inclusion be excited by an effective exciting field $\mathbf{u}^e$ Then Eq. (28) can be approximately replaced by

$$\mathbf{u}(\mathbf{r}; \mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_N) = \mathbf{u}^i(\mathbf{r}) + \sum_{k=1}^{N} \mathbf{T}^s(r_k)\mathbf{u}^e(\mathbf{r}|\mathbf{r}_k; \mathbf{r}_1, \ldots{}' \ldots, \mathbf{r}_N). \tag{29}$$

After performing configurational average over Eq. (29), we obtain

$$\langle \mathbf{u}(\mathbf{r}; \mathbf{r}_1, \ldots, \mathbf{r}_N) \rangle = \mathbf{u}^i(\mathbf{r}) + \int n(\mathbf{r}_k)\mathbf{T}^s(\mathbf{r}_k)\langle \mathbf{u}^e(\mathbf{r}|\mathbf{r}_k; \mathbf{r}_1, \ldots{}' \ldots, \mathbf{r}_N)\rangle \, \mathrm{d}V_k. \tag{30}$$

The quantity $\langle \mathbf{u}^e(\mathbf{r}|\mathbf{r}_k; \mathbf{r}_1, \ldots{}' \ldots, \mathbf{r}_N)\rangle$ represents the exciting field acting on the kth scatterer averaged over all possible configurations of the other scatterers. Therefore it is in fact the counterpart of the averaged total field with one inclusion absent. The averaged total field with N inclusions involved, $\langle \mathbf{u}(\mathbf{r}; \mathbf{r}_1, \ldots, \mathbf{r}_N)\rangle$, would differ from the averaged total field with one less inclusion involved only by terms of order $1/N$. As the number of inclusions increases, we may make the self-consistent approximation

$$\langle \mathbf{u}_k^e(\mathbf{r}|\mathbf{r}_k; \mathbf{r}_1, \ldots{}' \ldots, \mathbf{r}_N)\rangle \approx \langle \mathbf{u}(\mathbf{r}; \mathbf{r}_1, \ldots, \mathbf{r}_N)\rangle. \tag{31}$$

This approximation was proposed first by Foldy (1945) and later modified by Lax (1952) by introducing a correction parameter. Then, we obtain the integral equation.

$$\begin{aligned}\langle \mathbf{u}(\mathbf{r}; \mathbf{r}_1, \ldots, \mathbf{r}_N)\rangle &= \mathbf{u}^i(\mathbf{r}) + \int n(\mathbf{r}_k)\mathbf{T}^s(\mathbf{r}_k)\langle \mathbf{u}(\mathbf{r}; \mathbf{r}_1, \ldots, \mathbf{r}_N)\rangle \, \mathrm{d}V_k \\ &= \mathbf{u}^i(\mathbf{r}) + \int n(\mathbf{r}_k)\mathbf{G}\overline{T}^s(\mathbf{r}_k) \, \mathrm{d}V_k \langle \mathbf{u}(\mathbf{r}; \mathbf{r}_1, \ldots, \mathbf{r}_N)\rangle.\end{aligned} \tag{32}$$

This equation can be rewritten in a compact form

$$\langle \mathbf{u} \rangle = \mathbf{u}^i + n\mathbf{G}\overline{\mathbf{T}}^s\langle \mathbf{u} \rangle, \tag{33}$$

where $\mathbf{T}^s$ is fractionalized by $\mathbf{T}^s = \mathbf{G}\overline{\mathbf{T}}^s$ and $\overline{\mathbf{T}}^s$ is still called the scattering operator. $\mathbf{G}$ is the Green function tensor of the homogeneous and isotropic host medium, which satisfies

$$(\lambda_0 + \mu_0)\nabla(\nabla \cdot \mathbf{G}) + \mu_0 \nabla^2 \mathbf{G} - \rho_0 \omega^2 \mathbf{G} + \mathbf{I}\delta(\mathbf{r} - \mathbf{r}') = 0, \tag{34}$$

or in an operator form

$$\mathbf{L}^0\mathbf{G} + \mathbf{I} = \mathbf{0}, \tag{35}$$

where the operator $\mathbf{L}^{\mathbf{0}}$ is written as

$$L_{ij}^0(k) = (\lambda_0 + \mu_0)k_i k_j + (\mu_0 k^2 - \rho_0 \omega^2)\delta_{ij}. \tag{36}$$

For a homogeneous and isotropic medium, the equation of motion is expressed as

$$\mathbf{L}^0\mathbf{u} = \mathbf{0}, \tag{37}$$

and the dispersion relation can be obtained from

$$\mathbf{u}^*\mathbf{L}^0\mathbf{u} = 0. \tag{38}$$

For the longitudinal and shear waves, Eq. (38) leads to

$$k_{p0}^2 = \frac{\rho_0\omega^2}{\lambda_0 + 2\mu_0}, \tag{39a}$$

$$k_{s0}^2 = \frac{\rho_0\omega^2}{\mu_0}, \tag{39b}$$

where k_{p0} and k_{s0} are the wave numbers of P and S waves propagating through the homogeneous medium, respectively. After performing wave operator $\mathbf{L^0}$ on both sides of Eq. (33), we obtain the dispersion relations of the effective longitudinal and shear waves propagating through the composite material

$$k_{p^*}^2 = k_{p0}^2 + (\lambda_0 + 2\mu_0)^{-1} n\overline{T}_{ij}^s a_i a_j, \tag{40a}$$

$$k_{s^*}^2 = k_{s0}^2 + \mu_0^{-1} n\overline{T}_{ij}^s b_i b_j, \tag{40b}$$

where k_{p^*} and k_{s^*} are the effective wave numbers of P and S waves through the composite material, respectively. $\mathbf{a}$ and $\mathbf{b}$ are polarization vectors of P and S waves, respectively. It was proved by Devaney (1980) that the scattering operator $\overline{\mathbf{T}}^s$ of a single inclusion bears a simple relation to the far-field scattering amplitude vectors of the waves scattered by the inclusion

$$\overline{T}_{ij}^s a_i a_j = \frac{4\pi\rho_0\omega^2}{k_{p0}^2}[\mathbf{F}_p(k_{p^*}, k_{s^*}) \cdot \mathbf{a}]\big|_{\theta=0}, \tag{41a}$$

$$\overline{T}_{ij}^s b_i b_j = \frac{4\pi\rho_0\omega^2}{k_{s0}^2}[\mathbf{F}_s(k_{p^*}, k_{s^*}) \cdot \mathbf{b}]\big|_{\theta=0}, \tag{41b}$$

where $\mathbf{F}_p(k_{p^*}, k_{s^*})$ and $\mathbf{F}_s(k_{p^*}, k_{s^*})$ are the far-field scattering amplitude vectors of P and S waves, respectively. By inserting Eq. (41) into Eq. (40), we obtain the desired dispersion relations of the averaged waves in term of the far-field scattering amplitude vectors of a single inclusion.

$$k_{p^*}^2 = k_{p0}^2 + 4\pi n[\mathbf{F}_p(k_{p^*}, k_{s^*}) \cdot \mathbf{a}]\big|_{\theta=0}, \tag{42a}$$

$$k_{s^*}^2 = k_{s0}^2 + 4\pi n[\mathbf{F}_s(k_{p^*}, k_{s^*}) \cdot \mathbf{b}]\big|_{\theta=0}, \tag{42b}$$

where $\theta = 0$ denotes the incident direction.

The propagation of a scalar wave through an inhomogeneous medium with distributed scatterers was studied early by Foldy (1945) and the wave number of the coherent plane wave was given by

$$\left(\frac{k_*}{k_0}\right)^2 = 1 + \frac{4\pi n}{k_0^2} f(k_0), \tag{43}$$

where k_* is the wave number of the coherent wave and k_0 is that of the incident wave. $f(k_0)$ is the isotropic scattering amplitude of the potential scattered by a single inclusion embedded in a homogenous medium. Lax (1952) modified Foldy's treatment by introducing a correction parameter c' (a measure of the ratio of the effective exciting field $\langle \mathbf{u}^e \rangle$ to the macroscopic average field $\langle \mathbf{u} \rangle$) and extend to the anisotropic scattering by replacing $f(k_0)$ with the forward scattering amplitude $f(k_0, 0)$

$$\left(\frac{k_*}{k_0}\right)^2 = 1 + \frac{4\pi n}{k_0^2} c' f(k_0, 0). \tag{44}$$

Waterman and Truell (1961) provided an alternative formula based on the double plane wave theory, in which the backward scattering amplitude $f(k_0, \pi)$ was considered,

$$\left(\frac{k_*}{k_0}\right)^2 = \left[1 + \frac{2\pi n}{k_0^2} f(k_0, 0)\right]^2 - \left[\frac{2\pi n}{k_0^2} f(k_0, \pi)\right]^2, \tag{45}$$

By only retaining the chain-scattering (or neglecting the shuttle-scattering) in the multiple scattering process, Twersky obtained (Ishimaru, 1978)

$$k_* = k_0 + \frac{2\pi n}{k_0} f(k_0, 0). \tag{46}$$

Moreover, based on the independent scattering approximation Gubernatis (1984) obtained

$$k_*^2 = k_0^2 + 4\pi n f(k_0, 0). \tag{47}$$

A comparison among these equations can be made, and it is noted that Waterman and Truell's equation reduces to Foldy's equation for the isotropic scattering where $f(k_0, 0) = f(k_0, \pi)$, to Twersky's equation when the backward scattering amplitude is neglected, and to Gubernatis's equation when the second rank terms of the number density are neglected. In Eq. (42) the far-field scattered amplitude vectors of displacement, $\mathbf{F}_p(k_{p^*}, k_{s^*})$ and $\mathbf{F}_s(k_{p^*}, k_{s^*})$, are included, and the effective wave numbers are expressed in an implicit form. In order to simplify the computation $\mathbf{F}_p(k_{p^*}, k_{s^*})$ and $\mathbf{F}_s(k_{p^*}, k_{s^*})$ can be approximated by $\mathbf{F}_p(k_{p0}, k_{s0})$ and $\mathbf{F}_s(k_{p0}, k_{s0})$ for the case of weak scattering. Because $\mathbf{F}_p(k_{p0}, k_{s0})$ and $\mathbf{F}_s(k_{p0}, k_{s0})$ are, in general, complex-valued and frequency-dependent even for the elastic interphase, the effective wave numbers of P and S waves, k_{p^*} and k_{s^*}, are thus complex-valued and frequency-dependent. The real part of the complex-valued wave number is related to the phase velocity and the imaginary part represents the attenuation of waves

$$k_{p^*}(\omega) = k_{p^*}^r(\omega) + \mathrm{i}k_{p^*}^i(\omega) = \omega/c_p^* + \mathrm{i}\alpha_p^*, \tag{48a}$$

$$k_{s^*}(\omega) = k_{s^*}^r(\omega) + \mathrm{i}k_{s^*}^i(\omega) = \omega/c_s^* + \mathrm{i}\alpha_s^*, \tag{48b}$$

where, c_p^* and c_s^* are the phase velocities, and α_p^* and α_s^* are the attenuation of P and S waves, respectively. Further, the real and imaginary parts of the complex-valued wave number are not independent but connected by the non-local Kramer–Kronig relations (Donnell et al., 1981)

$$k_*^r(\omega) = \frac{2}{\pi} \mathrm{p.v.} \int_0^\infty \frac{\omega' k_*^i(\omega)}{\omega'^2 - \omega^2} \, \mathrm{d}\omega', \tag{49a}$$

$$k_*^i(\omega) = -\frac{2}{\pi} \mathrm{p.v.} \int_0^\infty \frac{\omega k_*^r(\omega')}{\omega'^2 - \omega^2} \, \mathrm{d}\omega', \tag{49b}$$

where 'p.v.' denotes the principal value integral. The complex-valued propagation constants mean a wave propagating with attenuation. In other word, the effective or average waves propagating through an inhomogeneous composite medium will be attenuated due to the multiple scattering among the inclusions. On the other hand, waves propagating through a dissipative medium are of complex-valued wave numbers too but due to the energy absorption. In the present case of the presence of the viscoelastic interphase, the complex-valued wave numbers of the coherent waves result from both energy diffusion and energy absorption. The effective elastic moduli of the composites can be obtained from the effective wave number by

$$\mu_*(\omega) = \mu_*^r(\omega) + \mathrm{i}\mu_*^i(\omega) = \mu_0 \frac{\rho_*}{\rho_0}\left(\frac{k_{s0}}{k_{s^*}}\right)^2, \tag{50a}$$

$$\lambda_*(\omega) = \lambda_*^r(\omega) + \mathrm{i}\lambda_*^i(\omega) = (\lambda_0 + 2\mu_0)\frac{\rho_*}{\rho_0}\left(\frac{k_{p0}}{k_{p^*}}\right)^2 - 2\mu_0\frac{\rho_*}{\rho_0}\left(\frac{k_{s0}}{k_{s^*}}\right)^2, \tag{50b}$$

where the effective density of the composites, ρ_*, can be obtained straightforward from the volume average

$$\rho_* = (1 - c - c\bar{c})\rho_0 + c\rho_1 + c\bar{c}\rho_2, \tag{51}$$

where $\bar{c} = 3(h/a) + 3(h/a)^2 + (h/a)^3$.

4. Numerical results and discussion

The effective properties of a SiC–Al composite material will be predicted in this section. The mechanical properties of the constituents are given in Table 1. In order to examine the influence of the viscoelastic interphase on the dynamic effective properties, for example, the effective phase velocities, the effective attenuation and the effective elastic moduli, a thin homogeneous interphase between the inclusion and the matrix is introduced. The standard linear solid model of a viscoelastic material is used to described the mechanical behavior of the viscoelastic interphase. The isotropic relaxation functions and their corresponding complex moduli are

$$\mu_2(t) = \mu_{21} + (\mu_{20} - \mu_{21})\exp(-t/\tau_\mu), \tag{52a}$$

$$\mu_2^v(\omega) = \mathrm{i}\omega F(\mu_2(t)) = (\mu_{21} - \mathrm{i}\mu_{20}\omega\tau_\mu)/(1 - \mathrm{i}\omega\tau_\mu), \tag{52b}$$

$$\lambda_2(t) = \lambda_{21} + (\lambda_{20} - \lambda_{21})\exp(-t/\tau_\lambda), \tag{53a}$$

$$\lambda_2^v(\omega) = \mathrm{i}\omega F(\lambda_2(t)) = (\lambda_{21} - \mathrm{i}\lambda_{20}\omega\tau_\lambda)/(1 - \mathrm{i}\omega\tau_\lambda), \tag{53b}$$

where μ_{20} and λ_{20} are the short-term (or initial) moduli, and μ_{21} and λ_{21} are the long-term (or final) moduli of the viscoelastic interphase, respectively. τ_λ and τ_μ are the relaxation times. It is noted that the elastic interphase can be recovered by letting $\lambda_{20} = \lambda_{21}$ and $\mu_{20} = \mu_{21}$ or $\tau_\lambda = \tau_\mu = \infty$. In this numerical example, the initial moduli, μ_{20} and λ_{20}, are assumed to be the average of the elastic moduli of the inclusion and the matrix, respectively. Two relaxation times, i.e. $\tau_\lambda = \tau_\mu = 1.0\mathrm{E}{-4}(\mathrm{s})$ and $4.0\mathrm{E}{-5}(\mathrm{s})$ with $\mu_{20}/\mu_{21} = \lambda_{20}/\lambda_{21} = 5$ are considered. It should be pointed out that the choice of these material constants is mainly for

Table 1
Material constants of SiC and Al

Materials	λ (GPa)	μ (GPa)	ρ (kg/m^3)	c_p (km/s)	c_s (km/s)
SiC	98.0	188.1	3181	12.21	7.69
Al	57.5	26.5	2706	6.39	3.129

From Shindo et al. (1995).

the sake of demonstration. It can be seen in Fig. 3 that the viscous effects of such viscoelastic materials are notable at the range of frequency $0.1 < k_{s0}a < 5$.

In Fig. 2, the predicted effective wave numbers (k_{p^*}, k_{s^*}) from Eqs. (45)–(47), namely, Waterman and Truell's, Twersky's and Gubernatis's equations, are compared. It can be seen that Twersky's equation, in which the backscattering amplitude is neglected, underestimates the phase velocities and the attenuation. However, the deviation from the results obtained from Waterman and Truell's equations decreases gradually with the increasing frequency. This is because the back scattering amplitude, in general, becomes smaller when the frequency increases. The results obtained from Gubernatis's equation give a good approximation to that obtained from Waterman and Truell's equation at the volume concentration considered. But it can be predicted that the deviation will increase with the increasing volume concentration because the second rank terms of the volume concentration are no longer insignificant. If $\mathbf{F}_p(k_{p^*}, k_{s^*})$ and $\mathbf{F}_s(k_{p^*}, k_{s^*})$ are approximated by $\mathbf{F}_p(k_{p0}, k_{s0})$ and $\mathbf{F}_s(k_{p0}, k_{s0})$, the results obtained from Eq. (42) are the same with that obtained from Gubernatis's equation. In addition, the numerical results obtained from the present program based on Eq. (42) with $\mathbf{F}_p(k_{p^*}, k_{s^*})$ and $\mathbf{F}_s(k_{p^*}, k_{s^*})$ approximated by $\mathbf{F}_p(k_{p0}, k_{s0})$ and $\mathbf{F}_s(k_{p0}, k_{s0})$ are the same with that obtained by Shindo et al. (1995). Hence the validity of the Fortran codes in the present study for evaluating the effective wave numbers is verified.

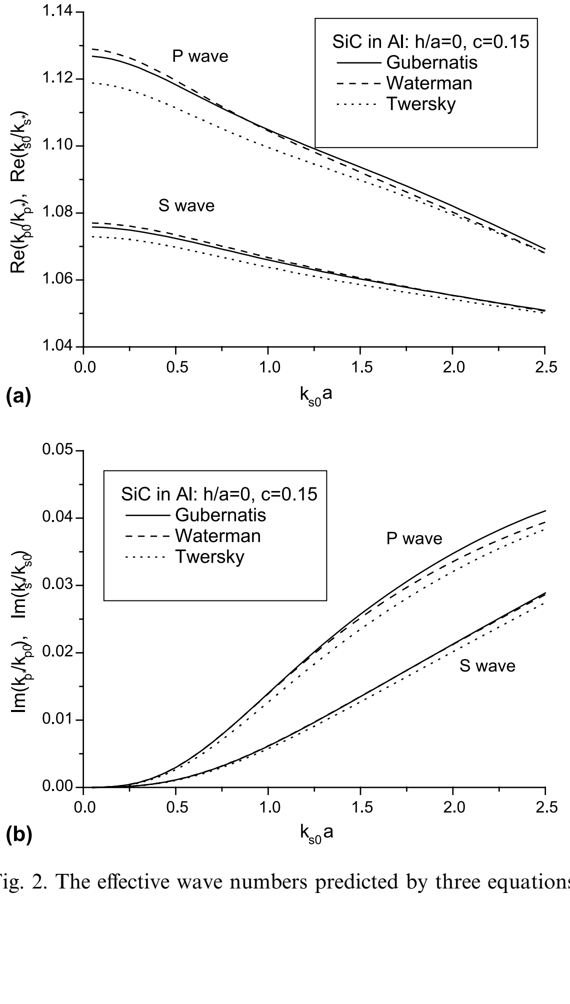

Fig. 2. The effective wave numbers predicted by three equations.

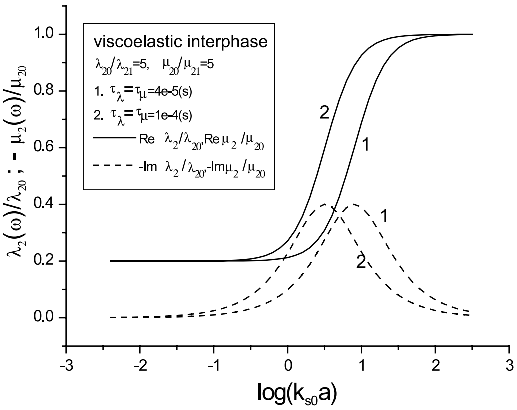

Fig. 3. The frequency-dependent complex moduli of a viscoelastic interphase.

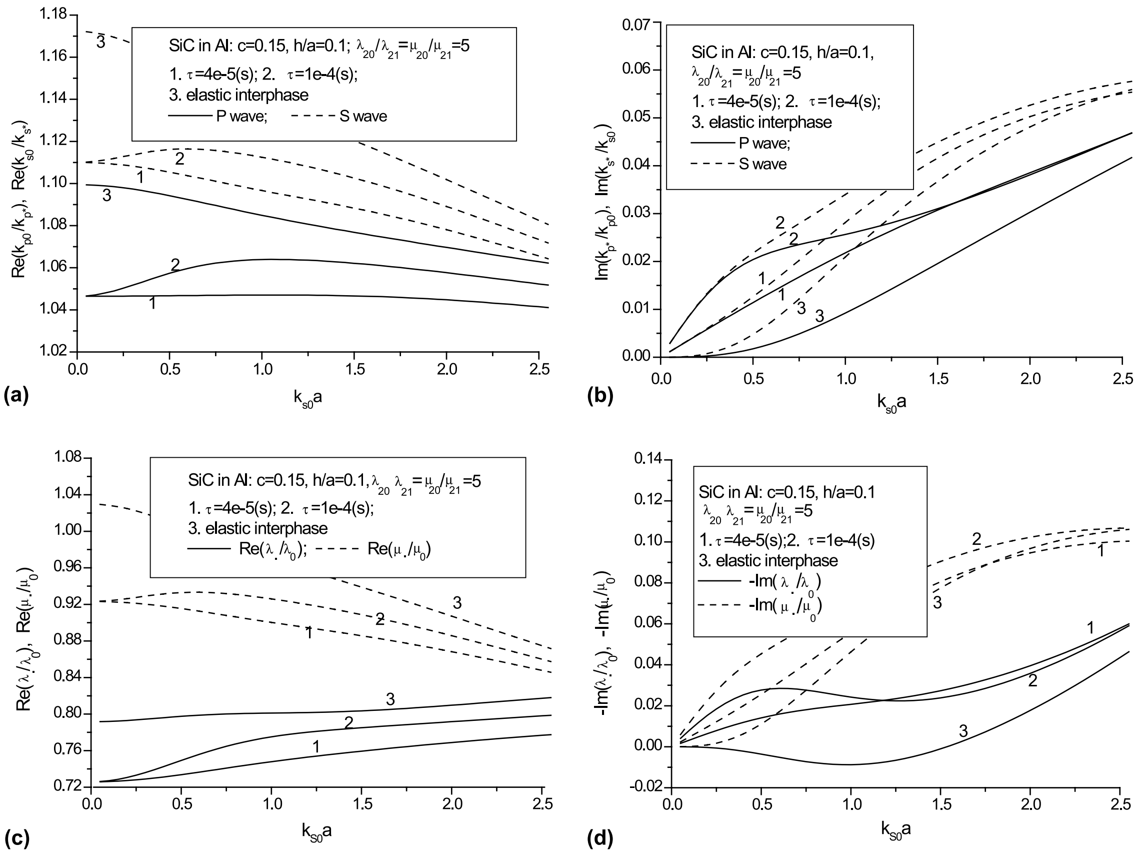

Fig. 4. The normalized effective wave numbers, attenuations and elastic moduli of the composite SiC–Al with an elastic or viscoelastic interphase ($\tau = \tau_\lambda = \tau_\mu$).

The frequency-dependent complex moduli of the viscoelastic interphase are shown in Fig. 3. The imaginary parts of the complex moduli are related to the viscosity of the material, which is restricted to a finite frequency range if the standard linear solid model is used. The viscoelastic interphase reduces to the elastic interphase with the long-term modulus at a relatively low frequency, whereas to the elastic interphase with the short-term modulus at a relatively high frequency. The predicted effective phase velocities and attenuations are shown in Fig. 4(a) and (b). It is noted that the phase velocities decrease but the attenuations increase for both P and S waves at a relatively low frequency due to the viscous effect of the interphase. However, the viscosity effect of the interphase decreases gradually with the increase of the frequency. The predicted effective muduli are shown in Fig. 4(c) and (d). It can be seen that the viscosity of the interphase can affect the effective moduli significantly at a relatively low frequency but the effect of the viscosity decreases gradually when the frequency increases. The viscous effect of the interphase on the imaginary parts of complex-valued elastic moduli is similar with that on the attenuation. From Fig. 4(b) and (c), it is can be seen that the attenuation tends to zero as the frequency tends to zero for both elastic and viscoelastic interphases. For two kinds of interphases with the same initial and final moduli but the different relaxation times, the deviation between them will vanish at a relatively low and high frequency, and reach maximum at a specific moderate frequency. This means that we can change the mechanical properties of composites at an interesting frequency range through the design of the interphase. On the other hand, the initial and final moduli can affect the mechanical properties of composites at the initial and final stages of loading. For example, if $\lambda_{21} = \mu_{21} = 0$, the particle-reinforced composite behaves in the final stage as a porous material after sufficient relaxation of the interphase. It should be also noted that the dissipative nature of the interphase and the multiple scattering effects of the distributed inclusions are both contributed to the attenuation of waves. But the mechanisms of the attenuation in these two cases are different distinctively. The dissipative nature results in the energy absorption but the multiple scattering effect only results in energy diffusion. The numerical results in the present study show that the viscous effect of the interphase dominates at a relatively low frequency but the multiple scattering effect dominates gradually with the increase of the frequency.

5. Concluding remarks

The forward scattering amplitudes are of importance in predicting the effective properties of a composite material. And the backward scattering amplitude is less important especially in a relative high frequency. The forward scattering amplitudes are related closely to the mechanical behavior of the interphase between the matrix and the inclusion, thus the interphase can affect the effective properties of the composite reinforced by the distributed inclusions significantly. This means that the desired effective properties of a composite can be obtained by an appropriate design of the interphase. An interphase of viscoelastic properties can be introduced to provide the relaxation and damping characteristics to an otherwise elastic brittle composite. The numerical results show that the viscous effects of the interphase can depress the effective phase velocities and the effective elastic moduli, but boost the effective attenuation. The initial and the final elastic moduli of the interphase can change the mechanical properties of the composite at the initial and the final stages of loading. And the mechanical properties of the composite at a specific interested frequency range can be changed by the aborative selection of a specific relaxation time. Moreover, both of the dissipative nature of the interphase and the multiple scattering effect of the distributed inclusions are contributed to the attenuation of the coherent waves. But the mechanisms of the attenuation in these two cases are different distinctively. The dissipative nature of the interphase results in the energy absorption but the multiple scattering effect results in the energy diffusion only. Furthermore, the viscous effect of the interphase dominates at a relatively low frequency and the multiple scattering effect dominates at a relatively high frequency.

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